Rayat Shikshan Sanstha's

Karmaveer Bhaurao Patil College Vashi, Navi Mumbai

Autonomous College

[University of Mumbai]

Syllabus for Approval

Sr. No.	Heading	Particulars
1	Title of Course	M.Sc. I Mathematics
2	Eligibility for Admission	T.Y.B.Sc. (Mathematics), From a recognized university
3	Passing Marks	40%
4	Ordinances/Regulations (if any)	
5	No. of Years/Semesters	One year/Two semester
6	Level	P.G.
7	Pattern	Semester
8	Status	New
9	To be implemented from Academic year	2018-2019

AC- 01/09/2018

Item No-2.27

Rayat Shikshan Sanstha's
KARMAVEER BHAURAO PATIL COLLEGE, VASHI. NAVI MUMBAI
(AUTONOMOUS COLLEGE)

Sector-15- A, Vashi, Navi Mumbai - 400 703

Syllabus for M.Sc. I Mathematics

Program: M.Sc.

Course: M.Sc. I Mathematics

(Choice Based Credit, Grading and Semester System with effect from the academic year 2018-2019)

Preamble of the Syllabus:

Master of Science (M.Sc.) in Mathematics is a post graduation programme of Department of Mathematics, Karmaveer Bhaurao Patil College Vashi, Navi Mumbai [Autonomous College]

The Choice Based Credit and Grading System to be implemented through this curriculum, would allow students to develop a strong footing in the fundamentals and specialize in the disciplines of his/her liking and abilities. The students pursuing this course would have to develop understanding of various aspects of the mathematics. The conceptual understanding, development of experimental skills, developing the aptitude for academic and professional skills, acquiring basic concepts and understanding of hyphenated techniques are among such important aspects.

Rayat Shikshan Sanstha's
KARMAVEER BHAURAO PATIL COLLEGE, VASHI
[AUTONOMOUS COLLEGE]

Department of Mathematics
M. Sc. Mathematics

Program Specific Outcomes

At the end of the Programme, the students will be able to:

1. Apply knowledge of mathematics, in all the fields of learning including higher research and its extension.
2. Innovate, invent and solve complex mathematical problems using the knowledge of pure and applied mathematics.
3. Solve one dimensional wave and heat equations employing the methods in partial differential equations.
4. Explain the knowledge of contemporary issues in the field of Mathematics and applied sciences.
5. Work effectively as an individual, and also as a member or a leader in multi-linguistic and multi-disciplinary teams.
6. Adjust themselves completely to the demands of the growing field of Mathematics by lifelong learning.
7. Effectively communicate about their field of expertise on their activities, with their peer and society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentation.
8. Crack lectureship and fellowship exams approved by UGC like CSIR- NET and SET.

Rayat Shikshan Sanstha's

KARMAVEER BHAURAO PATIL COLLEGE, VASHI

[AUTONOMOUS COLLEGE]

Department of Mathematics

M.Sc. Mathematics
Choice Based Credit System (CBCS)

SEMESTER – I

Course Code	Unit	Topic	Credit	L/W
Algebra I				
PGMT101	I	Dual spaces	5	4
	II	Determinants, eigen values and eigen vectors		
	III	Invariant Subspaces and applications		
	IV	Bilinear forms		
Analysis I				
PGMT102	I	Euclidean space $\mathbb{R}^n$	5	4
	II	Riemann integration		
	III	Differentiable functions		
	IV	Inverse function theorem , Implicit function theorem		
Complex Analysis				
PGMT103	I	Holomorphic functions	5	4
	II	Contour integration, Cauchy-Goursat theorem		
	III	Holomorphic functions and their properties		
	IV	Isolated singularities, Conformal Mappings and multivalued Functions		
Discrete Mathematics				
PGMT104	I	Number Theory	5	4
	II	Advanced Counting		
	III	Recurrence relations		
	IV	Polya's theory of counting		
Set Theory and Logic				
PGMT105	I	Introduction to logic and sets	4	4
	II	Sets and functions		
	III	Partial order		
	IV	Lattices		

SEMESTER II

Course Code	Unit	Topic	Credit	L/W
Algebra II				
PGMT201	I	Groups, group homomorphisms	5	4
	II	Groups acting on sets, Sylow’s theorems		
	III	Rings, Fields		
	IV	Divisibility in integral domains, finite fields		
Topology				
PGMT202	I	Topological spaces	5	4
	II	Connected and Compact topological spaces		
	III	Countability and Separation Axioms		
	IV	Complete metric spaces		
Differential Geometry				
PGMT203	I	Geometry of $\mathbb{R}^n$	5	4
	II	Plane and Space Curves		
	III	Regular Surfaces		
	IV	Curvature of surfaces		
Differential Equations				
PGMT204	I	Picard's theorem	5	4
	II	Linear Ordinary Differential Equations		
	III	Series solutions and Sturm Liouville’s theory		
	IV	Fourier series		
Probability Theory				
PGMT205	I	Basics of Probability	4	4
	II	Probability measure		
	III	Random variables		
	IV	Limit theorems		

Teaching Pattern for Semester I and II:

1. Four lectures per week per course. Each lecture is of 60 minutes duration.

2. In addition, there shall be tutorials, seminars as necessary for each of the five courses.

SEMESTER I

PGMT101: ALGEBRA I

(All Results have to be done with proof unless otherwise stated).

Unit I. Dual spaces (15 Lectures)

Learning Outcomes:

1. Recall the basic concepts of vector space over a field, linear transformation and matrix representation of a linear transformation.
2. Find dual basis for finite dimensional vector spaces.
3. Explain the relation between matrices representing a linear transformation and its transpose.

Content of the unit:

(Review) Vector spaces over a field, linear independence, basis and dimension, infinite dimensional vector spaces. Linear transformations, kernel and image, relationship of linear transformations with matrices, invertible linear transformations, rank-nullity theorem (for finite dimensional vector spaces), application: characterization of an isomorphism from a finite-dimensional vector space to itself. (No question be asked)
Dual spaces of a vector space, dual basis (for finite dimensional vector spaces), Double dual V^{**} of a Vector space V and canonical embedding of V into V^{**}. Isomorphism of V and its double dual in the finite-dimensional case. Transpose T^t of a linear transformation T, relation between matrices representing T and T^t.

Unit II. Determinants, Eigen values and Eigenvectors (15 Lectures)

Learning Outcomes:

1. Evaluate determinant using Laplace expansion, product and transposes and determinant of a linear transformation.
2. Determine the characteristic polynomial, Eigen value and Eigen vectors and apply to find the minimal polynomial of a matrix.

Content of the unit:

Determinants as an alternating multilinear map, existence and uniqueness, Laplace expansion of determinant, determinants of products and transposes, determinants and invertible linear transformations, determinant of a linear transformation.

Eigen values and Eigen vectors, Characteristic polynomial, Minimal polynomial, Triangulable and diagonalizable linear operators. Matrix limits and Markov chains. Application of Stochastic Matrices: Google Page Rank Algorithm

Unit III. Invariant Subspaces and Applications (15 Lectures)

Learning Outcomes:

1. Apply Cayley Hamilton theorem to find the inverse of a matrix.
2. Compute minimal polynomials and Jordan Canonical forms for nilpotent matrices.
3. Solve system of linear differential equations using Jordan Canonical form.

Content of the unit:

Invariant subspaces and Cayley-Hamilton theorem. Nilpotent linear transformations on finite dimensional vector spaces, computations of Minimum polynomials and Jordan Canonical Forms for nilpotent matrices, Jordan canonical forms in general through examples.

The derivative operator on $\mathbb{C}^{\infty}$, the space of infinitely differentiable functions, polynomial operators, solution space as a subspace of $\mathbb{C}^{\infty}$, Solution to an equation of order 1, the general case and connection with the auxiliary polynomial.

Application of Jordan Canonical Form in solving system of linear differentiable equations.

Unit IV: Bilinear forms (15 Lectures)

Learning Outcomes:

1. Recall inner product spaces, orthonormal basis.
2. Use different operators like normal, self-adjoint and symmetric operators.

Content of the unit:

(Review) Inner product spaces, orthonormal basis. (No question be asked)
Adjoint of a linear operator, normal and self-adjoint operators, unitary operators and orthogonal operators and their matrices, orthogonal projections and the spectral theorem.
Applications: Singular Value Decomposition. Bilinear and quadratic forms, Sylvester's Law.
Least squares method as an application of orthogonal projection. Application of SVD to least square solutions and Image Compression.

Recommended Text Books:

1. S.H.Friedberg, A. J. Insel, L.E.Spence: Linear Algebra, 4th Ed. Prentice-Hall.
2. S. Kumaresan: Linear Algebra, a Geometric Approach, Prentice-Hall.
3. David Lay: Linear Algebra and Applications.
4. Gilbert Strang: Introduction to Linear Algebra, Wellesley-Cambridge Pres.
5. Steven G. Krantz: Jordan Canonical Form Theory and Practice.
6. Serge Lang: Linear Algebra, Springer-Verlag Undergraduate Text in Mathematics.

PGMT102: ANALYSIS I

Unit I. Euclidean space $\mathbb{R}^n$ (15 Lectures)
Learning Outcomes:

1. Recall Inner product space, norm linear space and vector space.
2. Define Euclidean space $\mathbb{R}^n$.
3. Distinguish among open and closed sets on different topologies of $\mathbb{R}^n$.
4. Verify Cauchy-Schwarz inequality for two vectors of $\mathbb{R}^n$.

Content of the unit:

Euclidean space $\mathbb{R}^n$: inner product $\langle x, y\rangle = \sum_{j=1}^{n}$ xjyj of $x = (x_1, x_2, \dots \dots x_n)$ $y = (y_1, y_2, \dots \dots y_n) \in \mathbb{R}^n$ and properties , norm $\|x\| = \sqrt{\sum_{j=1}^{n} x_j^2}$ of $x = (x_1, x_2, \dots \dots x_n) \in \mathbb{R}^n$, Cauchy-Schwarz inequality, properties of the norm function $\|x\|\ of\ \mathbb{R}^n$ (ref: [4] W. Rudin or [5] M. Spivak)

Standard topology on $\mathbb{R}^n$: open subsets of $\mathbb{R}^n$, closed subsets of $\mathbb{R}^n$, interior A^0 and boundary ∂A of a subset A of $\mathbb{R}^n$: (ref: [5] M. Spivak)

Operator norm $\|T\|$of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ ($\|T\| = \sup\{\|T(v)\|: v \in \mathbb{R}^n \ \&\ \|v\| \le 1\}$) and its properties such as: For all linear maps $S, T : \mathbb{R}^n \to \mathbb{R}^m$ and $R : \mathbb{R}^m \to \mathbb{R}^k$

1. $\|S + T\| \le \|S\| + \|T\|$
2. $\|RoS\| \le \|R\|\|S\|$
3. $\|cT\| = |c|\|T\|, c \in \mathbb{R}$

(Ref: [1] C.C. Pugh or [2] A. Browder)
Compactness: Open cover of a subset of $\mathbb{R}^n$, Compact subsets of $\mathbb{R}^n$ (A subset K of $\mathbb{R}^n$ is compact if every open cover of K contains a finite subcover), Heine-Borel theorem (statement only), the Cartesian product of two compact subsets of $\mathbb{R}^n$ is compact (statement only), every closed and bounded subset of $\mathbb{R}^n$ is compact. Bolzano-Weierstrass theorem: Any bounded sequence in $\mathbb{R}^n$ has a converging subsequence.

Brief review of following three topics:

1. Functions and Continuity: Notation: $A \subset \mathbb{R}^n$ arbitrary non-empty set. A function $f: A \to \mathbb{R}^m$ and its component functions, continuity of a function ($\in -\delta$; definition). A function $f: A \to \mathbb{R}^m$ is continuous if and only if for every open subset $V \subset \mathbb{R}^m$there is an open subset U of $\mathbb{R}^n$ such that $f^{-1}(V) = A \cap U$.
2. Continuity and compactness: Let $K \subset \mathbb{R}^n$ be a compact subset and f: $K \to \mathbb{R}^m$ be any continuous function. Then f is uniformly continuous, and f(K) is a compact subset of $\mathbb{R}^m$
3. Continuity and connectedness: Connected subsets of are intervals. If $f:E \to \mathbb{R}$ is continuous where $E \subset R^n$ and E is connected , then $f(E) \subset \mathbb{R}$ is connected.

Unit II: Riemann Integration (15 Lectures)

Learning Outcomes:

1. Determine Riemann integrability of a function using definition and Riemann criteria.
2. List the properties of Riemann integrable functions.
3. Apply Fubini's theorem to evaluate double integrals.

Content of the unit:

Riemann Integration over a rectangle in $\mathbb{R}^n$; Riemann Integrable functions, Continuous functions are Riemann integrable, Measure zero sets, Lebesgues Theorem (statement only), Fubini's Theorem and applications.
Reference for Unit II: M. Spivak, Calculus on Manifolds.

Unit III: Differentiable functions (15 Lectures)
Learning Outcomes:

1. Define differentiability of a vector valued function.
2. Evaluate total derivative of a differentiable vector valued function.
3. Apply Chain rule to evaluate the derivative of a composite function.
4. List the results on total derivative.

Content of the unit:

Differentiable functions on $\mathbb{R}^n$, the total derivative $(Df)_p$ of a differentiable function f : U→ $\mathbb{R}^m$ at p ∈ U where U is open in$\mathbb{R}^n$; uniqueness of total derivative, differentiability implies continuity. (ref: [1] C.C. Pugh or [2] A. Browder)

Chain rule, Applications of chain rule such as:

1. Let γ be a differentiable curve in an open subset U of $\mathbb{R}^n$: Let f : U U→ $\mathbb{R}$ be a differentiable function and let g(t) = f(γ (t)). Then g′(t) = $\langle(\nabla f(\gamma(t), \gamma'(t)\rangle$.
2. Computation of total derivatives of real valued functions such as
   (a) the determinant function det(X), X ∈M_n ($\mathbb{R}$).
   (b) the Euclidean inner product function $\langle x, y\rangle$, (x, y) ∈ $\mathbb{R}^n \times \mathbb{R}^n$

(ref: [5] M. Spivak & [4] W. Rudin)

Results on total derivative:

1. If f : $\mathbb{R}^n \rightarrow \mathbb{R}^m$is a constant function, then $(Df)_p = 0$ ∀ p ∈ $\mathbb{R}^n$
2. If f : $\mathbb{R}^n \rightarrow \mathbb{R}^m$ is a linear map, then $(Df)_p$ = f ∀ p ∈ $\mathbb{R}^n$
3. A function f = $(f_1,f_2,\ldots, f_m)$: $\mathbb{R}^n \rightarrow \mathbb{R}^m$ is differentiable at p ∈ $\mathbb{R}^n$ if and only if each f_j is differentiable at p ∈ $\mathbb{R}^n$; and $(Df)_p = ((Df_1)_p ,(Df_2)_p,\ldots, (Df_m)_p)$.

(ref: [5] M. Spivak)

Partial derivatives, directional derivative $(D_uf)(p)$ of a function f at p in the direction of the unit vector, Jacobian matrix, Jacobian determinant. Results:

1. If the total derivative of a map f = $(f_1, \ldots , f_m)$: U→ $\mathbb{R}^m$ (U open subset of $\mathbb{R}^n$) exists at p ∈ U; then all the partial derivatives $\frac{\partial f_j}{\partial x_j}$ exist at p
2. If all the partial derivatives $\frac{\partial f_j}{\partial x_j}$ of a map f = $(f_1, f_2,\ldots, f_m)$: U→ $\mathbb{R}^m$ (U open subset of $\mathbb{R}^n$) exist and are continuous on U, then f is differentiable.

(ref:[4] W. Rudin)
Derivatives of higher order, C^k-functions, C^∞-functions. (ref: [3] T. Apostol)

Unit IV: Inverse function theorem, Implicit function theorem (15 Lectures)

Learning Outcomes:

1. Demonstrate a working knowledge of Taylor's theorem, mean value inequality and mean value theorem.
2. Find stationary points, saddle points, maxima and minima of a differentiable function by applying second derivative test.
3. Apply inverse function theorem to evaluate the derivative of inverse function.

Content of the unit:

Theorem (Mean Value Inequality): Suppose f : U→ $\mathbb{R}^m$ is differentiable on an open subset U of $\mathbb{R}^n$and there is a real number such that $\|(Df)_x\| \leq M$ ∀ x ∈ U. If the segment [p, q] is contained in U; then $\|f(q) - f(p)\| \leq M\|q-p\|$ (ref: [1] C.C. Pugh or [2] A. Browder)
Mean Value Theorem: Let f : U→ $\mathbb{R}^m$ is differentiable on an open subset U of $\mathbb{R}^n$. Let p, q ∈ U such that the segment [p; q] is contained in U. Then for every vector $v \in \mathbb{R}^n$ there is a point $x \in$ [p, q] such that $\langle v, f(q) - f(p)\rangle = \langle v, (Df)_x(q-p)\rangle\rangle$. (ref: [3] T. Apostol)
If $f : U \rightarrow \mathbb{R}^m$ is differentiable on a connected open subset U of $\mathbb{R}^n$ and $(Df)_x = 0$ ∀ x ∈ U, then f is a constant map.

Taylor expansion for a real valued C^m-function defined on an open subset of $\mathbb{R}^n$, stationary points (critical points), maxima, minima, saddle points, second derivative test for extrema at a stationary point of a real valued C^2-function defined on an open subset of $\mathbb{R}^n$. (ref: [3] T. Apostol)

Contraction mapping theorem. Inverse function theorem , Implicit function theorem. (ref: [2] A. Browder)

Recommended Text Books

1. C.C. Pugh: Real mathematical analysis, Springer UTM.
2. A. Browder: Mathematical Analysis, An Introduction, Springer.
3. T. Apostol: Mathematical Analysis, Narosa.
4. W. Rudin: Principles of Mathematical Analysis, Mcgraw-Hill India.
5. M. Spivak: Calculus on Manifolds, Harper-Collins Publishers.

PGMT103: COMPLEX ANALYSIS

Unit I: Holomorphic functions (15 Lectures)
Learning Outcomes:

1. Represent complex numbers algebraically and geometrically.
2. Define and analyze limits and continuity for functions of complex variables.
3. Understand about the analytic functions and Cauchy-Riemann equations.
4. Analyze sequences and series of analytic functions and types of convergence.

Content of the unit:

Review: Complex Numbers, Geometry of the complex plane, Riemann sphere, Complex sequences and series, Sequences and series of functions in $\mathbb{C}$, Weierstrass's M-test, Uniform convergence, (no questions be asked).
Complex differentiable functions, Cauchy-Riemann equations, A complex differentiable function defined on an open subset of $\mathbb{C}$ is called a Holomorphic function.
Ratio test and root test for convergence of a series of complex numbers. Complex Power series, radius of convergence of a power series, Cauchy-Hadamard formula for radius of convergence of a power series. Examples of convergent power series such as exponential series, cosine series and sine series, and the basic properties of the functions e^z, cos z, sin z.
Abel's theorem: Let $\sum_{n\geq 0} a_n(z-z_0)^n$ be a power series, of radius of convergence R > 0. Then the function f defined by f(z) $=\sum \ a_n(z-z_0)^n$ is holomorphic on the open ball $|z-z_0| < R$ and $f'(z) = \sum_{n\geq 1} na_n(z-z_0)^{n-1} \ \forall |z-z_0| < R$.
Applications of Abel's theorem such as $exp'(z) = exp\, z, cos'(z) = -\,sin\, z, sin'(z) = cos\, z\ (z \in \mathbb{C})$
Chain Rule. A basic result: Let Ω_1, Ω_2 be open subsets of $\mathbb{C}$. Suppose $f : \Omega_1 \to \mathbb{C}$ is a Holomorphic function with $f'(z) \neq 0\ \forall\ z \in \Omega_1$ and $g : \Omega_2 \to \mathbb{C}$ be a continuous function such that $g(\Omega_2) \subset \Omega_1$ and $f(g(w)) = w\ \forall\ w \in \Omega_2$. Then g is a holomorphic function on $\Omega_2$and $g'(w) = \frac{1}{f'(g(w))}\ \ \forall\ w \in \Omega_2$.

Unit II: Contour integration, Cauchy-Goursat theorem (15 Lectures)
Learning Outcomes:

1. State Cauchy-Goursat theorem for a rectangular and triangular region and use it to evaluate complex integrals over a closed contour.
2. Define homomorphic functions and write down the power series representation of such functions.

Content of the unit:

Contour integration, Cauchy-Goursat Theorem for a rectangular region or a triangular region. Primitives. Existence of primitives: If f is Holomorphic on a disc U, then it has a primitive on U and the integral of f along any closed contour in U is 0. Local Cauchy's Formula for discs, Power series representation of Holomorphic functions, Cauchy's estimates, Cauchy's theorem (homotopy version)

Unit III: Holomorphic functions and their properties (15 Lectures)
Learning Outcomes:

1. Understand about the entire functions including the fundamental theorem of algebra.
2. Calculate the winding number of a closed curve.
3. Evaluate complex contour integrals and apply the Cauchy integral theorem in its various versions.

Content of the unit:

Entire functions, Liouville's theorem, Morera's theorem, the Fundamental theorem of Algebra. The index (winding number) of a closed curve, Cauchy integral formula. Zeros of Holomorphic functions, Identity theorem. Counting zeros; Open Mapping Theorem, Maximum modulus theorem.

Unit IV: Isolated singularities, Conformal Mappings and multivalued Functions (15 Lectures)
Learning Outcomes:

1. Represent functions as Taylor and Laurent series.
2. Classify singularities and poles, find residues and evaluate complex integrals using the residue theorem.
3. Understand the conformal mapping.

Content of the unit:

Isolated singularities: removable singularities and Removable singularity theorem, poles and essential singularities. Laurent Series development. Casorati-Weierstrass's theorem

Residue Theorem and evaluation of standard types of integrals by the residue calculus method. Conformal mappings. If $f : G \to \mathbb{C}$ is a holomorphic function on the open subset G of $\mathbb{C}$ and $f'(z) \neq 0 \ \forall z \in G$. then f is a conformal map. Mobius transformations (fractional linear transformation or linear transformation). Any Mobius transformation which fixes three distinct points is necessarily the identity map. Cross ratio (z_1, z_2, z_3, z_4) of four points z_1, z_2, z_3, z_4. Cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points z_1, z_2, z_3, z_4 lie on a circle.
Multivalued Functions: $\sqrt{Z}$, the logarithm as the inverse of exponential, branches of logarithm, the principal branch ln(z) of the logarithmic function on $\mathbb{C} - \{z \in \mathbb{C}: z \leq 0\}$ is a Holomorphic function and $ln'(z) = \frac{1}{z}$

Recommended Text Books:

1. J. B. Conway, Functions of one Complex variable, Springer.
2. R. Remmert: Theory of complex functions, Springer.
3. A. R. Shastri: An introduction to complex analysis, Macmillan.
4. J. W. Brown and R. V. Churchill : Complex variables and applications
5. L. V. Ahlfors: Complex analysis, McGraw Hill .
6. Donold Sarason: Notes on Complex function theory, Hindustan book agency.

7. Steven Krantz: Complex analysis: The geometric view point, second edition, carus mathematical monographs

PGMT 104: DISCRETE MATHEMATICS

Unit I. Number theory (15 Lectures)
Learning Outcomes:

1. Use Cardona's equation to evaluate two roots of a cubic equation.
2. Use Chinese reminder theorem to solve n-congruence equations.
3. Check the solvability of linear Diophantine equations in 2 and 3 variables using divisibility.
4. Explain the types of occupancy problems.

Content of the unit:

Divisibility, Linear Diophantine equations, Cardano's Method, Congruences, Quadratic residues, Arithmetic functions,
Types of occupancy problems, distribution of distinguishable and indistinguishable objects into distinguishable and indistinguishable boxes (with condition on distribution) Stirling numbers of second and first kind. Selections with Repetitions.

Unit II. Advanced counting (15 Lectures)
Learning Outcomes:

1. Apply pigeonhole principle in different problems of interest.
2. Apply Inclusion exclusion principle in different problems of interest.
3. Explain the concept of permutations with forbidden positions and with the help of rook polynomial calculate the number of arrangements of rook in forbidden positions.

Content of the unit:

Pigeon-hole principle, generalized pigeon-hole principle and its applications, Erdos- Szekers theorem on monotone subsequences, A theorem of Ramsey. Inclusion-Exclusion Principle and its applications. Derangement. Permutations with Forbidden Positions, Restricted Positions and Rook Polynomials.

Unit III. Recurrence Relations (15 Lectures)
Learning Outcomes:

1. Learn how to work with some of the discrete structures which include sets, relations, functions, graphs and recurrence relations.
2. Demonstrate a working knowledge of Recurrence relations and Generating functions in different applied fields.

Content of the unit:

The Fibonacci sequence, Linear homogeneous recurrence relations with constant coefficient. Proof of the solution in case of distinct roots and statement of the theorem giving a general solution (in case of repeated roots), Iteration and Induction. Ordinary generating Functions, Exponential Generating Functions, algebraic manipulations with power series, generating functions for counting combinations with and without repetitions, exponential generating function for bell numbers, applications to counting, use of generating functions for solving recurrence relations.

Unit IV. Polyas Theory of counting (15 Lectures)
Learning Outcomes:

1. State orbit stabilizer theorem and use it to prove Burnside lemma.
2. Deduce that the set of orbits forms an equivalence relation.
3. Write Polyas enumeration formula and use it in different problems of interest.

Content of the unit:

Equivalence relations and orbits under a permutation group action. Orbit stabilizer theorem, Burnside Lemma and its applications, Cycle index, Polyas Formula, Applications of Polyas Formula.

Recommended Text Books

1. D. M. Burton, Introduction to Number Theory, McGraw-Hill.
2. Nadkarni and Telang, Introduction to Number Theory
3. V. Krishnamurthy:Combinatorics: Theory and applications, A liated East-West Press.
4. Richard A. Brualdi: Introductory Combinatorics, Pearson.
5. A. Tucker: Applied Combinatorics, John Wiley & Sons.
6. Norman L. Biggs: Discrete Mathematics, Oxford University Press.
7. Kenneth Rosen: Discrete Mathematics and its applications, Tata McGraw Hills.
8. Sharad S. Sane, Combinatorial Techniques, Hindustan Book Agency, 2013.

PGMT105: SET THEORY AND LOGIC

(All results have to be done with proof unless otherwise stated.)

Unit I. Introduction to logic and sets (15 Lectures)
Learning Outcomes:

1. Define sets and use the representation of a given set to distinguish membership properties of elements, subsets and perform its operations.
2. Determine the truth value of quantified sentences, given its universal set by constructing truth value table or by applying the concept of solution sets.
3. Learn about the logical foundations of mathematical concepts such as number and set.

Content of the unit:

Statements and their negations using quantifiers (including examples from Linear Algebra, Basic Real Analysis and Elementary Number Theory), Truth value, Logical connectives and Truth tables, Conditional statements, Logical inferences, Methods of proof with examples.

Basic Set theory: Union, intersection and complement, indexed sets, the algebra of sets, power set, Cartesian product, relations, equivalence relations, partitions, equivalence classes and transversals with examples.

Unit II. Sets and functions (15 Lectures)
Learning Outcomes:

1. Define functions, composition of functions and explain when two functions are said to be equal.
2. Classify different types of functions. Draw graphs of various functions.
3. Explain countable, finite and uncountable sets with examples.
4. Find cardinalities of various sets.

Content of the unit:

Functions, composition of functions, surjections, injections, bijections, inverse functions, Equality of functions, Graph of functions, Image of sets under functions, Inverse images of sets under functions, Cantor's theorem, Schroeder-Bernstein theorem, Finite Countable and Uncountable sets, Cardinality of sets, Cardinalities of $\mathbb{N}$, $\mathbb{N} \times \mathbb{N}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{R} \times \mathbb{R}$ etc.

Unit III. Partial order (15 Lectures)
Learning Outcomes:

1. Define partially ordered and totally ordered set with examples.
2. Use principle of mathematical induction to prove various results.
3. Explain Russell's paradox with the help of example.
4. Apply Zorn's lemma to find maximal ideals of a non-trivial ring with unity.

Content of the unit:

Order relations, order types, partial order, total order, well ordered sets, Principle of Mathematical Inductions, Well-ordering Principle, Equivalence of induction principles and well-ordering relation, Russell's paradox, Chains, bounds and maximal elements, Statements of the Axiom of Choice, Zorn's lemma, applications of Zorn's lemma to maximal ideals and to bases of vector spaces.

Unit IV. Lattices (15 Lectures)
Learning Outcomes:

1. Draw Hasse diagram of partially ordered set.
2. Understand the concept of lattices, distributive and modular lattices.
3. Understand how lattices and Boolean algebra are used as tools and mathematical models in the study of networks.

Content of the unit:

Mobius inversion formula on a partially ordered set, Hasse Diagrams of a partially ordered set, Lattices, Distributive and Modular Lattices, complements, Boolean Algebra, Boolean expressions, Only elementary Applications.

Recommended Text Books:

1. Ajit Kumar, S. Kumaresan and B. K. Sarma, Foundation Course in Mathematics, Narosa.
2. Robert R. Stoll: Set theory and logic, Freeman & Co.
3. James Munkres: Topology, Prentice-Hall India;
4. Richard A. Brualdi: Introductory Combinatorics, Pearson.
5. Kenneth Rosen: Discrete Mathematics and its applications, Tata McGraw Hills.
6. Larry J. Gerstein: Introduction to mathematical structures and proofs, Springer.
7. Robert Wolf: Proof, logic and conjecture, the mathematician's toolbox, W. H. Freemon.

SEMESTER II
PGMT201: ALGEBRA II
(All results have to be done with proof unless otherwise stated.)

Unit I. Groups, group Homomorphisms (15 lectures)
Learning Outcomes:

1. Explore the properties of groups, sub-groups, including symmetric groups, permutation groups, cyclic groups, normal subgroups and quotient groups.
2. Understand the concepts of homomorphism and isomorphism between groups.
3. Design, analyse and implement the concepts of homomorphism and isomorphism between groups for solving different types of problems, for example, isomorphism theorems, quotient groups etc.

Content of the unit:

Review: Groups, subgroups, normal subgroups, products of subgroups H and K, various cases depending on normality of H and K, center Z(G) of a group. Homomorphisms and kernels. Cyclic groups, Permutation groups, Dihedral groups, Matrix groups, the group of units U_n of $\mathbb{Z}_n$, Lagrange's theorem. (No questions to be asked).
Quotient groups. First isomorphism theorem and examples: quotients of groups of non-zero complex numbers, $Gl_n(\mathbb{R})$, real numbers by integers. Second and third isomorphism theorems for groups, applications. Automorphisms of a group. Automorphisms of cyclic groups. Inner automorphisms of a group. Product of groups. $\mathbb{Z}_{mn}$ as a product, Structure theorem of abelian groups and applications.

Unit II. Groups acting on sets, Sylow theorems (15 lectures)
Learning Outcomes:

1. Apply class equation and Sylow theorems to solve different problems.
2. Find centre of various groups.
3. Classify groups of small orders.
4. Explain the orbit stabiliser theorem, Cayley's theorem.

Content of the unit:

Center of a group, centralizer or normalizer of an element, conjugacy class C(a) of a in G.
Groups acting on sets, Examples: action of G on itself by conjugation, and by left multiplication on itself, and on the set of the left cosets of a subgroup. Centralizers, Normalizers, Orbits and Stabilizers, Cayley's Theorem, Class equation, Cauchy's theorem, p-groups, Commutativity of groups of order p^2, centre of a group of order p^n, Sylow's theorems and applications. Groups of order 15, 6. Semi-direct products, groups of order 12. Classification of Groups of small orders. Burnside Counting as an application of group action.

Unit III. Rings, Fields (15 lectures)
Learning Outcomes:

1. Explore the properties of rings, sub-rings, ideals including integral domain.
2. Design, analyse and implement the concepts of homomorphism and isomorphism between rings for solving different types of problems, for example, isomorphism theorems, quotient rings etc.
3. Explain the notion of an extension of a field, algebraic elements.
4. Design, analyse and implement the concepts of Gauss lemma, Eisenstein's irreducibility criterion, mod p irreducibility test.

Content of the unit:

Review: Rings (with unity), ideals, quotient rings, prime ideals, maximal ideals, ring homomorphisms, characteristic of a ring, isomorphism theorems for rings, relation between ideals in the ring and a quotient ring. Integral domains and their quotient fields. (no questions be asked).
Definition of field, characteristic of a field, subfields and prime subfields. Polynomial rings over a field F, irreducible polynomials over F. Prime, and maximal ideals in F[X], and their generators, unique factorization for polynomials over a field.
Definition of field extension, algebraic elements, minimal polynomial of an algebraic element, extension of a field obtained by adjoining one algebraic element. Kronecker's theorem, an application of Kronecker's theorem: Existence of a splitting field of a polynomial.

Unit IV. Divisibility in integral domains, finite fields (15 lectures)
Learning Outcomes:

1. Describe the structure of finite fields with specific examples.
2. Do computations in specific examples of finite fields.
3. Explore the properties of principle ideal domain, Euclidean ring, and Euclidean domain and Unique Factorization domain.
4. Utilize the polynomial rings, UFD, ED, PID to solve different related problems.

Content of the unit:

Prime elements, irreducible elements, Unique Factorization Domains, Principle Ideal Domains, Gauss's lemma, $\mathbb{Z}$, F[X] are UFD, irreducibility criterion, Eisenstein's criterion, Euclidean domains. Examples of: domain but not a UFD, UFD but not a PID, PID but not a Euclidean domain.
Finite integral domains are fields, Finite fields, order, existence of polynomials with no roots. Existence of a field of characteristic p with p^n elements for and prime p and positive integer n.

Recommended Text Books

1. Michael Artin: Algebra, Prentice-Hall India.
2. David Dummit, Richard Foote: Abstract Algebra, Wiley-India.
3. R.B.J.T. Allenby: Rings, Fields and Groups, An Introduction to Abstract Algebra, Elsevier (Indian edition).
4. J. B. Fraleigh, A first Course in Abstract Algebra, Narosa.
5. G. Santhanam, Algebra, Narosa

PGMT202: TOPOLOGY

Unit I. Topological spaces (15 Lectures)
Learning Outcomes:

1. Identify precisely when a collection of subsets of a given set equipped with a topology forms a topological space.
2. Understand the concepts of topological spaces and basic definitions of open sets, neighbourhood, interior, exterior, closure and their axioms.
3. Understand the concept of basis and subbasis, create new topological spaces by using subspace.

Content of the unit:

Topological spaces, basis, sub-basis, product topology (finite factors only), subspace topology, closure, interior, continuous functions, T_1, T_2 spaces, quotient topology.

Unit II. Connected and Compact topological spaces (15 Lectures)
Learning Outcomes:

1. Understand continuity, compactness and connectedness.
2. Find the connected components and path components of a topological space.
3. Study of theorems on connectedness and compactness.
4. Understand path connectedness with examples.

Content of the unit:

Connected topological spaces, path-connected topological spaces, continuity and connectedness, connected components of a topological space, Path components of a topological space.
Compact spaces, limit point compact spaces, continuity and compactness, tube lemma, compactness and product topology (finite factors only), local compactness, one point compactification.

Unit III. Countability and Separation Axioms (15 Lectures)
Learning Outcomes:

1. State the first, second countability and separable axioms. List the results based on first and second countability.
2. Explain the terms regular, normal and Lindelöf space.
3. Study of the properties of a regular space etc.

Content of the unit:

Countability Axioms, Separation Axioms, Separable spaces, Lindelöf spaces, Second countable spaces. A compact T_2 space is regular and normal space.

Unit IV. Complete metric spaces (15 Lectures)
Learning Outcomes:

1. Understand the concepts of complete metric space, completion of a metric space, total boundedness, compactness in a metric space and uniform continuity.
2. Observe the relation between Bolzano-Weierstrass property and sequentially compact set.
3. Give examples of topological space that are complete.

Content of the unit:

Complete metric spaces, Completion of a metric space, total boundedness, compactness in Metric spaces, sequentially compact metric spaces, uniform continuity, Lebesgue covering lemma, Arzela-Ascoli theorem.

Recommended Text Books

1. James Munkres: Topology, Pearson.
2. George Simmons: Topology and Modern Analysis, Tata Mcgraw-Hill.
3. M.A.Armstrong: Basic Topology, Springer UTM.
4. K.D.Joshi: General Topology

PGMT203: DIFFERENTIAL GEOMETRY

NOTE: 1. All results have to be done with proof unless otherwise stated.
2. Plenty of examples have to be solved.

Unit I. Geometry of $\mathbb{R}^n$ (15 Lectures)
Learning Outcomes:

1. Define inner product in $\mathbb{R}^n$, orthonormal basis, orthogonal transformations, orthogonal matrices.
2. Derive the equations of lines and planes in $\mathbb{R}^n$ and their parametric equations.
3. Classify rotations as elements of the groups $SO(2)$, $SO(3)$.
4. Review Inverse function theorem and implicit function theorem.

Content of the unit:

Inner product in $\mathbb{R}^n$**,** Lines and planes in$\mathbb{R}^n$ and their Parametric equations, Orthonormal basis, Orthogonal transformations, Orthogonal matrices, Hyperplanes in $\mathbb{R}^n$**,** Reflections and rotations, classification of rotations as elements of the groups SO(2), SO(3), Isometries of $\mathbb{R}^n$- classification. Review of inverse mapping theorem and implicit function theorem.
Reference for Unit I: S. Kumaresan, Linear Algebra, A Geometric Approach

Unit II. Plane and Space Curves (15 Lectures)
Learning Outcomes:

1. Define curves in $\mathbb{R}^2$ and $\mathbb{R}^3$.
2. Find arc length parameterization of various curves.
3. Calculate curvature and signed curvature for plane curves and curvature and torsion for space curves.
4. Derive the Serret-Frenet equations.
5. State the Fundamental theorem for plane curves and fundamental theorem for space curves.

Content of the unit:

Regular curves in $\mathbb{R}^2$and $\mathbb{R}^3$, Arc length parametrization, curvature and signed curvature for plane curves. Fundamental theorem for plane curves, Curvature and torsion for space curves, Serret-Frenet equations, Fundamental theorem for space curves.

Unit III. Regular Surfaces (15 Lectures)
Learning Outcomes:

1. Explain the geometry of different types of surfaces.
2. Understand the concept of smooth surfaces and orientable surfaces.
3. Deduce that a Mobius strip is non orientable.

Content of the unit:

Regular surfaces in $\mathbb{R}^3$, local coordinates and atlas with examples, Surfaces as level sets, Surfaces as graphs, Surfaces of revolution. Tangent vectors and tangent space to a surface at a point, Smooth functions on a surface, Differential of a smooth function defined on a surface, Normal Vector, Orientable surfaces, the first fundamental form.

Unit IV. Curvature of Surfaces (15 Lectures)
Learning Outcomes:

1. Derive the coefficients of first fundamental form and second fundamental form.
2. Compute principal curvature and Gaussian curvature.
3. Utilize geodesics, all its related terms, properties and theorems.

Content of the unit:

The Gauss map, Shape operator, The second fundamental form, Principal curvatures, Euler's formula, Meusnier's Theorem, Normal curvature, Gaussian curvature and mean curvature, Computation of Gaussian curvature, Isometries of surfaces, Gauss's Theorem. Egregium (without proof), Geodesics - definition, properties, geodesics and isometries- show that great circles are geodesics in sphere.

Reference for Units II, III, IV:

1. M. DoCarno, Differential geometry of curves and surfaces, Princeton University Press, 1976.
2. S. Montiel and A. Ros, Curves and Surfaces, AMS Graduate Studies in Mathematics, 2009.
3. A. Pressley, Elementary Differential Geometry, Springer UTM, 2009.
4. J. Thorpe, Elementary Topics in Differential Geometry, Springer UTM, 2007.
5. Christian Baer, Differential Geometry, Cambridge University Press.

PGMT204: DIFFERENTIAL EQUATIONS

Unit I. Picard's Theorem (15 Lectures)
Learning Outcomes:

1. Know Picard's method of obtaining successive approximations of solutions of first order differential equations.
2. Apply the concept of reduction of an n^{th}order linear Ordinary Differential Equation to a system of first order Ordinary Differential Equation to find solution of an n^{th}order linear Ordinary Differential Equation.

Content of the unit:

Existence and Uniqueness of solutions to initial value problem of first order ODE- both autonomous, non-autonomous (Picard's Theorem), Approximations, system of first order Picard's scheme of successive linear ODE with constant coefficients and variable coefficients, reduction of an n-th order linear ODE to a system of first order ODE.

Unit II. Linear Ordinary Differential Equations (15 Lectures)
Learning Outcomes:

1. Know Picard's method of obtaining successive approximations of solutions of first order differential equations.
2. Apply the concept of reduction of an n^{th}order linear Ordinary Differential Equation to a system of first order Ordinary Differential Equation to find solution of an n^{th}order linear Ordinary Differential Equation.

Content of the unit:

Existence and uniqueness results for an n-th order linear ODE with constant coefficients and variable coefficients, linear dependence and independence of solutions of a homogeneous n-th order linear ODE, Wronskian matrix, Lagrange's Method (variation of parameters), algebraic properties of the space of solutions of a non-homogeneous n-th order linear ODE.

Unit III. Series solutions and Sturm Liouville theory (15 Lectures)
Learning Outcomes:

1. Apply concept of power series to solve differential equations.
2. Express Sturm – Liouville theory.
3. Understand oscillation properties of solution.

Content of the unit:

Solutions in the form of power series for second order linear equations of Legendre and Bessel, Legendre polynomials, Bessel functions. Sturm- Liouville Theory: Sturm- Liouville Separation and comparison Theorems, Oscillation properties of solutions.

Unit IV: Fourier series (15 lectures)
Learning Outcomes:

1. Find the Fourier series representation of a function of one variable.
2. Understand the properties of Fourier series and its complex form.
3. Understand how to express any periodic function as a Fourier series.
4. Be able to expand an odd or even function as a half-range cosine or sine Fourier series.

Content of the unit:

Eigenvalues and eigenfunctions of Sturm-Liouville Boundary Value Problem, the vibrating string. Orthogonality of eigen functions, Dirichlet's conditions, Fourier series expansion of periodic functions (period 2pi & arbitrary period), Complex form of Fourier series, Half range Fourier series, Nth partial sum of Fourier series, Bessel's inequality, Parseval's identity (over complex field).

Note: ODE stands for Ordinary Differential Equations and PDE stands for Partial Differential Equations.

Recommended Text Books:
1. Units I and II:
(a) E.A. Coddington, An introduction to Ordinary Differential Equations, Dover Publication INC.
(b) E.A. Codington, N. Levinson, Theory of Ordinary differential Equations, Tata McGraw-Hill, India.
(c) Hurewicz W., Lectures on ordinary differential equations, M.I.T. Press.
(d) Morris W. Hirsch and Stephen Smale, Differential Equations, Dynamical Systems, Linear Algebra, Elsevier.
2. Unit III and Unit IV:
(a) G.F. Simmons, Differential equations with applications and historical notes, McGraw-Hill international edition.
(b) G.F. Simmons, S.G.Krantz, Differential equations, Theory, Technique and Practice,

Walter Rudin series in advanced mathematics, 1^{st} edition.
(c) G.B.Folland, Fourier series and its applications, AMS

PGMT205: PROBABILITY THEORY

Unit I: Probability basics (15 Lectures)
Learning Outcomes:

1. Calculate probabilities by applying probability laws and theoretical results.
2. Derive expressions for measures such as the mean and variance of common probability distributions using calculus and algebra.
3. Prove and apply the independence of a family of sigma-fields or random variables.

Content of the unit:

Modelling Random Experiments: Introduction to probability, probability space, events.
Classical probability spaces: uniform probability measure, fields, finite fields, finitely additive probability, Inclusion-exclusion principle,σ - fields, σ - fields generated by a family of sets, σ- field of Borel sets, Limit superior and limit inferior for a sequence of events.

Unit II: Probability measure (15 Lectures)
Learning Outcomes:

1. Calculate probabilities for joint distributions including marginal and conditional probabilities.
2. Calculate probabilities of independent events.
3. Discuss about Lebesgue integral for non-negative Borel function assuming its construction.

Content of the unit:

Probability measure, Continuity of probabilities, First Borel-Cantelli lemma, Discussion of Lebesgue measure on σ - field of Borel subsets of assuming its existence, Discussion of Lebesgue integral for non-negative Borel functions assuming its construction.
Discrete and absolutely continuous probability measures, conditional probability, total probability formula, Bayes formula, Independent events.

Unit III. Random variables (15 Lectures)
Learning Outcomes:

1. Identify an appropriate probability distribution for a given discrete or continuous random variable.
2. Derive probability distributions of functions of random variables.
3. Calculate the covariance and correlation for jointly distributed random variables and determine whether they are independent.

Content of the unit:

Random variables, simple random variables, discrete and absolutely continuous random variables, distribution of a random variable, distribution function of a random variable, Bernoulli, Binomial, Poisson and Normal distributions, Independent random variables, Expectation and variance of random variables both discrete and absolutely continuous.

Unit IV. Limit Theorems (15 Lectures)
Learning Outcomes:

1. Learn weak laws and strong laws of large numbers.
2. Apply results from large-sample theory and the central limit theorem to approximate a sampling distribution.
3. Learn characteristics function, Chebyshev inequality with examples.

Content of the unit:

Conditional expectations and their properties, characteristic functions, examples, Higher moments examples, Chebyshev inequality, Weak law of large numbers, Convergence of random variables, Kolmogorov strong law of large numbers (statement only), Central limit theorem (statement only).

Recommended Text Books:

1. M. Capinski, Tomasz Zastawniak: Probability Through Problems.
2. J. F. Rosenthal: A First Look at Rigorous Probability Theory, World Scientific.
3. Kai Lai Chung, Farid AitSahlia: Elementary Probability Theory, Springer Verlag.

Scheme of Examination

In each semester, the performance of the learners shall be evaluated into two parts. The learner's performance in each course shall be assessed by Continuous Internal Assessment with 40 marks in the first part, by conducting the **Semester End Examinations** with 60 marks in the second part.

Continuous Internal Assessment of 40 marks:

Unit Test	Project Work	Total
20 Marks	20 Marks	40 Marks

Project Work:

The presentation of the project is to be made by the student in front of the committee appointed by the Head of the Department of Mathematics of the college. This committee shall have two members, possibly with one external referee. Each project shall have maximum of 05 (Five) students.

The Marks for the project are detailed below:

Contents of the project: 10 marks
Presentation of Project: 5 marks
Viva of the project: 5 marks
Total Marks= 20 per project per student.

Semester End Examination of 60 marks:

(i) Duration: - Examination shall be of **Two and Half hours** duration.

(ii) Theory Question Paper Pattern:-

1. There shall be five questions each of 12 marks.
2. On each unit there will be one question and the fifth one will be based on entire syllabus.
3. All questions shall be compulsory with internal choice within each question.
4. Each question may be subdivided into sub-questions a, b, c, d. and the allocation of marks depend on the weightage of the topic.
5. Each question will be of 24 marks when marks of all the sub-questions are added (including the options) in that question.

Questions		Marks
Q 1	Based on Unit I	12
Q 2	Based on Unit II	12
Q 3	Based on Unit III	12
Q 4	Based on Unit IV	12
Q 5	Based on Unit V	12
	Total Marks	60